Assignment: homework assignment (Written Homework 13). The relevant section for this ILO evaluation is problem 4, which is stated as follows.

4. Let $f(x, y) = 3\sqrt{x} + 2y^2$.

(a) Find the equation of the tangent plane to f at the point $(4, 3)$.

(b) Use your linearization from part (a) to approximate the values of f at the points $(4.2, 3.05)$ and $(4.5, 2)$.

(c) Compare the approximations form part (b) to the exact values of f at the points $(4.2, 3.05)$ and $(4.5, 2)$. Which approximation is more accurate? Explain why this should be expected.

SOLUTION:

(a) First, evaluate f at $(4, 3)$:

$$f(4, 3) = 3\sqrt{4} + 2(3)^2 = 24$$

we find the partial derivatives of f:

$$f_x(x, y) = \frac{3}{2\sqrt{x}} \qquad \text{and} \qquad f_y(x, y) = 4y.$$

Next, evaluate these partial derivatives at $(4, 3)$:

$$f_x(4, 3) = \frac{3}{2\sqrt{4}} = \frac{3}{4} \qquad \text{and} \qquad f_y(4, 3) = 4 \cdot 3 = 12.$$

Thus, the equation of the tangent plane to f at the point $(4, 3)$ is

$$z = 24 + \frac{3}{4}(x - 4) + 12(y - 3),$$

which simplifies to

$$z = \frac{3}{4}x + 12y - 15.$$

(b) The linearization from part (a) is $L(x, y) = \frac{3}{4}x + 12y - 15$. This gives the following approximations of f at the specified points:

$$f(4.2, 3.05) \approx L(4.2, 3.05) = \frac{3}{4}(4.2) + 12(3.05) - 15 = 24.75$$

$$f(4.5, 2) \approx L(4.5, 2) = \frac{3}{4}(4.5) + 12(2) - 15 = 12.375$$

(c) The actual value of $f(4.2, 3.05)$ is 24.7532, while the approximation is 24.75. This approximation appears to be quite accurate, as it differs from the actual value by only 0.0032.

The actual value of $f(4.5, 2)$ is 14.364, while the approximation is 12.375. This approximation does not seem to be particularly accurate, as it differs from the actual value by nearly 2.

Since the linearization is computed using the point $(4, 3)$, we should expect the approximation to be more accurate for points that are close to $(4, 3)$. This is what we see above, since since $(4.2, 3.05)$ is much closer to $(4, 3)$ than is $(4.5, 2)$. The approximation at $(4.2, 3.05)$ is much more accurate than the approximation at $(4.5, 2)$.