St. Olaf Mathematics Department

# Math Mess

Department of Mathematics
St. Olaf College
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## This Week's Mathematics Colloquiums

Title: The Politics of Exclusion / A Problem From the Game SET?
Speakers: Jennifer Quinn / Amelia Taylor
Time: Tuesday, February 18th, 1:30pm / Thursday, February 20th, 11:30am
Place: SC 182

### Tuesday's Colloquium

The Principle of Inclusion-Exclusion is a useful mathematical method for attacking identities where consecutive terms have alternating positive and negative signs. While a powerful tool, it has a tendency to obscure any relationships between the sets being considered. In this talk, Professor Quinn will use bijections between odd sets and even sets to prove the same identities. The approach is simpler, more direct, and more concrete. Is it better? You decide.

Jennifer Quinn truly feels that she has led a charmed life. She had the privilege of earning a B.A. from Williams College, an M.S. from the University of Illinois, Chicago, and a Ph.D. from the University of Wisconsin, Madison. She cannot imagine a professional career that doesn't include doing mathematics and teaching students. In 1993, she joined the faculty at Occidental College in Los Angeles where she currently chairs the Mathematics Department. She was recently appointed as co-editor-elect for Math Horizons Magazine with colleague Art Benjamin from Harvey Mudd College. In 2001, she was honored to receive the Distinguished Teaching Award for the Southern California section of the MAA. Her research interests include combinatorial proof, graph theory, and combinatorial matrix theory.

### Thursday's Colloquium

The game SET? requires players to look at a collection of cards and select three cards that form a set. The cards have different shapes, colors, shades, and numbers of objects on them, which we will call parameters. A set is formed when a player finds three cards such that for each parameter the three cards are equal or are all distinct. Many combinatorial questions arise naturally from this (continued on back) from this game. One of these questions has generated quite a bit of interest in the

mathematical community, but is not necessarily stated as a combinatorial question. Professor Taylor will (continued on back) explain this problem, its interpretation in enumerative geometry and in algebra, and talk about approaches to solving the problem in low dimensions. She will conclude by connecting the problem to joint research with Dave Bayer.

Amelia Taylor grew up in Colorado, went to St. Olaf as an undergraduate, and received her Ph.D. from the University of Kansas. She is currently a VIGRE Postdoc at Rutgers University, doing research in computational commutative algebra. When Amelia is not doing mathematics, she plays ultimate frisbee for New Jersey's Electric Mayhem and enjoys skiing.

## Internship in Costa Rica

A research internship is available at the La Selva Biological Station, located in lowland rainforest in Costa Rica, to assist with the development of a mathematical model of the mutualistic interaction between the forest understory plant Piper cenocladum and the ant Pheidole bicornis. The objective of the mathematical model is to make long-term predictions of the number of plants that acquire ant colonies and the number of herbivores and herbivory rates on plants with and without resident ants. The intern position duties will be divided roughly evenly between assisting with the solution of the model and conducting experiments in La Selva's primary forest reserve to estimate model parameters. For more information, contact Professor Michael Smith (smith@stolaf.edu). The closing date is March 5th, 2003.

## Last Week's Problem

The problem is best approached by reconstructing Pascal's Triangle using only 0s and 1s, with the rule 1+1=0. A little experimentation suggests that the number of odd numbers in any row is always a power of two. Digging deeper one discovers that if n<2^k, then there are twice as many odd numbers in the (n+2^k)-th row than in the n-th. From this it follows that the power of two we seek is determined by the number of 1s in the base 2 representation of n. 2003=11111010011 in base 2, so there are 2^8=256 odd numbers in the 2003rd row of Pascal's Triangle. This problem was solved by members of the Problem Solving Seminar, none of whom bothered to write it up. Another problem for Pascal's Triangle lovers follows.

## Problem of the Week

For any positive integers n and k, n>k, let d=gcd(n,k) (so d is the greatest common divisor of n and k). Prove that $\frac{d}{n}\binom{n}{k}$ is an integer.

** Please submit all solutions to David Molnar (molnar@stolaf.edu) by noon on Sunday.

If you would like to receive a copy of the Math Mess in your P.O. Box weekly, please e-mail Donna Brakke at brakke@stolaf.edu.

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