St. Olaf Mathematics Department

Math Mess

Department of Mathematics
St. Olaf College
Northfield, MN 55057

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This Week's Mathematics Colloquium

Title: Testing for Primes in Polynomial Time
Speaker: Amelia Taylor
Time: Tuesday, May 4th, 1:30 pm - refreshments at 1:15
Place: SC 182

This Week's Colloquium

Even if we aren't sure of the formal definition of a prime number, we all know 2, 3, 5, 7, and 11 are prime numbers. But what about $5 \cdot 2^{34224}+1$? Since the time of Euclid, mathematicians have been asking questions about how to determine if a number is prime. Many algorithms for this process have been proposed, but until recently they have either required exponential time to run, or were non-deterministic (using randomness and getting probabilistic results).

In the fall of 2002, three mathematicians from India produced the first deterministic, polynomial time algorithm for determining if a number is prime. But is this algorithm practical? Unfortunately, there are a number of non-deterministic algorithms that still do much better, though from a theoretical point of view this new algorithm is advantageous: it answers a big open question, and the mathematics it uses is very interesting.

I will give some of the history of this problem and talk about some of the more interesting basic abstract algebra used to get the result. The talk will be accessible to anyone, but part of it will be especially appealing to students who are in, or have taken, abstract algebra.

Amelia Taylor is a one of the newer members of the Department of Mathematics, Statistics and Computer Science, and she digs anything that mixes commutative algebra and computer science – like testing for primes. She's done a few things in her years, like talking to Jennifer Connelly for an hour one night in a bar in New York City and traveling to the World Games in Japan with Team USA Ultimate as UPA staff member. Did we mention that she also met Russell Crowe in that same bar? We'll save the rest for another math mess bio down the road.

Last Week's Problem

Five (very smart, but absent-minded) St. Olaf Math majors go to Budapest. There are 9 dishes served at the caf there, but astonishingly no-one packed a Hungarian-English dictionary, so they have no way of knowing which dish is which. Also, the food at the caf is served family-style, so they can order 5 dishes, and they'll get 5 dishes, but not in any particular order. They make a list of the 9 dishes (palacsintas, gulyas, sajtos szendvics es leves, …) and the 9 names on the menu, which we'll just call A, B, C, D, E, F, G, H, and I. They visit the caf three times, ordering 5 dishes each time, and by clever choices figure out which dish is which. How do they do it?

This was a popular problem, and we received correct solutions from **Carl Carlson '05**, **Chris Ebert '06**, **Philip Schulte '06**, **Sara Krohn '05**, and **Cliff Corzatt**, who, it should be noted, knows Hungarian, presumably disqualifying him. The key was in realizing that the five dishes ordered on a given day need not be distinct. Thus, one possible solution is to order on day 1, AABCD; on day 2, EEBFG, and on day 3, HHCFI. No two dishes will appear the same number of times on all three days, so it can be determined from these orders which dish is which. As Carl points out, this solution actually allows us to determine which is which among ten dishes; if presented with such a scenario, simply never order J.

Problem of the Week

A regular hexagon is divided into congruent equilateral triangles. The triangles are then covered by "lozenges", which are two congruent equilateral triangles stuck together, kind of like a domino, but not square. There are three different orientations possible for a lozenge. Prove that no matter how the hexagon is tiled, the same number of lozenges will appear in each of the three orientations.

If you want to get the Mess problems ahead of time, they will be sent out on Thursdays on Molnar's math-probsolv email alias. Let him know if you would like to be added to the alias.

*** Please submit all solutions by Wednesday at 5 o'clock to David Molnar by e-mail (molnar@stolaf.edu) or by placing them in his box at OMH 201.

If you would like to receive a copy of the Math Mess in your P.O. Box weekly, please e-mail Donna Brakke at brakke@stolaf.edu.

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