## MS CS

 Mess

Department of Mathematics, Statistics and Computer Science
St. Olaf College

May 9, 2005
Volume 33, No. 22

Northfield, MN 55057

## This Week's Colloquium

Title: A New Angle on Euler's Formula
Speaker: Kristin Camenga '97, graduate student at Cornell University
Time: Tuesday, May $10^{\text {th }} 1: 30 \mathrm{pm}$
(treats at 1:15)
D1nno. CC1 Q ?
You may have seen Euler's formula for threedimensional polyhedra: $\mathrm{V}-\mathrm{E}+\mathrm{F}=2$, where V is the number of vertices, E is the number of edges and F is the number of faces. For example, a cube has $\mathrm{V}=8, \mathrm{E}=12$ and $\mathrm{F}=6$, giving $8-12+6=2$. In fact, there are similar formulas for higher dimensional convex polyhedra, called polytopes. In these formulas we count the number of faces in each dimension, and then add and subtract them - getting a fixed number in each dimension!

But the generalization doesn't stop there! Remarkably similar formulas exist which "count" the solid angles of polytopes. To "count" an angle at a face, we figure out what fractions of directions we can look from the face and see the inside of the polytope. So, for a cube, the angle at a vertex is $1 / 8$, the angle at an edge is $1 / 4$ and the angle at a face is $1 / 2$. The grand finale shows why the formulas are related using probability and projections.
(This talk should be accessible to anyone who understands fractions, addition and subtraction. Toys will be provided for visualization.)

Kristin Camenga '97 is a graduate student in mathematics at Cornell University, hoping to complete a masters in education and a PhD in math in 2006. After graduating from St. Olaf and receiving a Fulbright scholarship to study graph theory and math education in Budapest, Hungary, she taught high school for two years in Delano, MN. She lives with her husband, Andrew, and her cat, Ink, in Bath, NY, and has made herself (in)famous in the Cornell math department for her willingness to make a fool of herself in the department Spring Concert.

## Some Math Aphor isms

I have hardly ever known a mathematician who was able to reason.

- Stephen Hawking

It is impossible to be a mathematician without being a poet in soul.

- Sofia Kovalevskaya

Mathematicians are born, not made.
Henri Poincaré
Belief is no substitute for arithmetic.
Henry Spencer
I must admit that outside the university, it is difficult to be a pure mathematician. No one in my family understands what I do. My neighbors wonder why I spend all my time in my study scribbling with pencil on a yellow pad of paper instead of going outside to mow the lawn.

## Mathematics professor Martha Wallace honor ed by MCTM

By Le Ann Finger '85

St. Olaf Professor of Mathematics Martha Wallace was honored by the Minnesota Council of Teachers of Mathematics (MCTM) -- the state organization of K-12 mathematics teachers. Wallace received an honorary membership -- equivalent to a "lifetime achievement" honor -- for her work within the organization.

Wallace served as president of the MCTM organization several years ago. She helped create the Minnesota Mathematics Mobilization (M3), the nation's first state coalition that enlisted leaders of education, business and government in support of mathematics education.
"Most colleges and universities prepare scientists, citizens and teachers -- three very different challenges-- to guide the next generation with expertise and enthusiasm," says Lynn Steen, professor of mathematics. "Martha has met all three of these challenges with consistent energy, practical imagination and unwavering common sense."

At St. Olaf, Wallace imagined, developed and sustained a pioneering "Visiting Master Teacher" program that brought experienced school teachers into the St. Olaf mathematics department to teach college mathematics courses, to update knowledge and skills in new areas, and to help the faculty learn about policy and pedagogy in K -12 education.

Wallace rose through the ranks to full professor in a mathematics department that is among the nation's leading producers of Ph.D. mathematicians. In addition, Wallace introduced and led a faculty education seminar in the mathematics department to compliment an ongoing faculty research seminar.

Wallace, a 1975 St. Olaf alumna, has been on the faculty since 1977.

## MCTM Conference

Six Olaf mathematics students spent the last weekend in April attending the Minnesota Council of Teachers of Mathematics Spring Conference in Duluth, and brought home lots of goodies that will help them in their teaching.

## Nate Earley, Mark Kingsbury, Sheila

 Slowinski, Liz Staloch, and Elizabeth White attended a pre-conference meeting just for new teachers, where they got advice from experienced math teachers, free books on teaching math and many math t shirts. Elizabeth won the major door prize, a Silver TI-84 that she can use for teaching high school math. The St. Olaf students continued to collect books and math toys at the conference on Friday and Saturday. Then, at the final session on the conference, Brian Strand, who had not been able to attend the pre-conference meeting, won the grand prize of $\$ 1000$ savings bond!The St. Olaf students rated the conference tops in getting them ready to teach mathematics. Ask them about their weekend! (And if Nate tries to gift you with a t -shirt, ask him where he got it.)

## Problem for the Summer

For what positive integers is $\mathrm{n}^{2}+\mathrm{n}+19$ a perfect square? Clearly $\mathrm{n}=2$ and $\mathrm{n}=5$ are solutions. Are there any more? Can you find a complete list of solutions?

## Last Week's Problem

This week's POW comes with 3 levels. Please send solutions to any and all levels that you solve. Credit will be given to anyone solving any of these questions.

First we warm-up for the second question. Is it true that given any six numbers between 1 and 10 , that
at least two of the given integers have to be relatively prime to each other (meaning their greatest common divisor is 1 )?

If you've got the first one, $\dot{s}$ it true that given at least $\mathrm{k} / 2$ integers between n and $\mathrm{n}+\mathrm{k}$ then at least two of the given integers have to be relatively prime to each other? [ $\mathrm{k} / 2$ may not be an integer, so I am assuming that the number of integers is the smallest integer bigger than $\mathrm{k} / 2$, or larger].

Bonus: prove that given at least $\mathrm{n} / 2$ integers (same comment from above applies) between 1 and $n$ that there must be at least 2 numbers a and b with the property that either a divides b or b divides a .

## Solution:

Congratulations to Robert Orme '05 for his solutions. The first answer is true and this first solution is Robert's. One way to approach it is to point out that since there are four primes and the number one, making a set of 5 , between 1 and $10(1,2,3,5,7)$, any six numbers chosen between 1 and 10 must include a prime or 1 , which is relatively prime to whatever other five numbers you pick. There is a more general argument using the pigeon hole principle. Any 6 numbers (i.e. more than half) between 1 and 10 will have to have 2 consecutive numbers in it and consecutive integers are always relatively prime (summer practice: prove this).

The second answer is false as stated. Let n and k be even. Then half of the numbers between n and $\mathrm{n}+\mathrm{k}$ are even. So we can pick our $\mathrm{k} / 2$ integers from the even numbers, which have a common factor of 2 and so are not relatively prime. Example: $\mathrm{n}=10, \mathrm{k}=6$. So $\mathrm{k} / 2=3$ and $\mathrm{n}+\mathrm{k}=16$. Pick 10,14 , and 16.

However, if we restate the question to make sure that the random set we are choosing has more than half the numbers between n and $\mathrm{n}+\mathrm{k}$ the argument for the warm-up, using the pigeonhole argument, still works to prove that in this case we must have two relatively prime.

We'll leave the bonus for the summer. Again, I believe that it is not true as stated, but is true if we make sure we have at least one more than half the number of integers between 1 and $n$.

## A Few More Thoughts

Belief is no substitute for arithmetic.
Henry Spencer
There are three kinds of lies: Lies, Damn Lies, and Statistics.
Benjamin Disraeli
A mathematician is a blind man in a dark room looking for a black cat which isn't there.
Charles Darwin
Mathematics commands all my respect, but I have no use for engines.

- Joseph Conrad

Mathematicians are a species of Frenchmen: if you say something to them they translate it into their own language and presto! It is something entirely different.

- J.W. Goethe
***If you would like to receive a copy of the Math Mess in your P.O. Box weekly, please e-mail Donna Brakke at brakke@stolaf.edu.


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