St. Olaf Mathematics Department

Math Mess

Department of Mathematics
St. Olaf College
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This Week's Mathematics Colloquium

Title: Experimental Design, Multiple Regression, and Flying Gummy Bears
Speaker: Paul Roback
Time: Tuesday, December 3rd, 1:30 pm
Place: SC 182

This Week's Colloquium

Among the significant questions perplexing scientists and engineers of the 21st century is this: how should one build a launcher to maximize the flying distance of gummy bears? We will apply principles of experimental design and multiple regression to collect and analyze data addressing this important question. First, we will discuss elements of study design (including blocking, randomization, and factorial crossing) which should lead to the collection of useful data; and, second, we will examine elements of multiple regression analysis (including indicator variables, interaction terms, and lack-of-fit tests) for generating data-based conclusions.

Paul Roback is currently an assistant professor at Connecticut College. After graduating from St. Olaf, Paul earned an MS from Iowa State and a PhD in Statistics from Colorado State. In between, Paul worked for three years as a clinical statistician at Eli Lilly. His academic interests include Bayesian statistics, computer-intensive methods, and ecological applications, and his personal interests include almost any sport.

NCS Contest Results

On the 16th of November 70 teams from 23 colleges and universities competed in the NCS team math problem-solving contest., both new highs. Impressively, all five St. Olaf teams finished in the top half, meaning everybody was above average! Leading the Oles were The Meat-lovers: **Jerad Parish, Jonathan Von Stroh, Michael Zahniser** who tied for 6th place. Other Ole teams competing were: Cauliflower: **Nick Maryns, Adam McDougall, Jason Saccomano** (11th), Xtra Cheese: **Joel Beard, Justin Fredenburg, and Dan Visscher** (23rd), Pineapple: **Janine Dahl, Noah Loome, Mark Schmelzle** (29th) and Green Peppers: **Elizabeth Johnson, Kyle Manley, Robert Orme**. (31st). Congratulations to everybody on their excellent performances!

Football Math

Given its field position, should a team punt, kick a field goal, or go for the first down? The problem has been analyzed mathematically by David Romer, an economist at Berkeley, using dynamic programming. An example: Your team has a fourth down on the 2-yard line. Do you take the easy 3 points from a field goal or go for a touchdown? The touchdown, with its almost automatic extra point, has probability 40% in that position; so its expected value, 2.8 points, is less than the expected value of a field goal. But, says Romer, the resulting field position for the opposing team must also be considered. He calculates (using data from 700 NFL games) that in a first quarter "the value of a first-and-10 on a team's own 1-yard line ... is -1.6 points." However, the kickoff after a field goal would leave them at the 27-yard line on the average, where a first-and-10 is worth +0.6 points according to the dynamic programming algorithm. This difference brings an extra 2.2 points to the expected value of the try for touchdown, and makes it clearly preferable to the field goal attempt.

["Strategies on Fourth Down, From a Mathematical Point of View," by Virginia Postrel, appeared in the September 12, 2002 New York Times. Summary taken from www.ams.org.]

Last Week's Problem

Show that the fraction with minimal denominator in the interval (3/5,5/8) is 8/13. In general, given two "adjacent" fractions a/b and c/d (adjacent means $|ad - bc| = 1$), how do you find the fraction with minimal denominator between a/b and c/d?

3/5 and 5/8 are adjacent fractions. To see that the fraction with minimal denominator between them is 8/13, it suffices to check by hand. It is in fact a general property of adjacent fractions that the interval between them contains no fractions of smaller denominator. To see why, check out anything about the "Farey tree," such as Conway and Guy's *The Book of Numbers*.

We received a solution from **Geoff Bolen '98** who points out that if we find a fraction m/n which is adjacent to **both** a/b and c/d, it will necessarily have denominator greater than max(b,d). Assume that a/b < c/d. Then applying the above fact to the two smaller intervals, there will be no fractions with denominator smaller than n in either the interval (a/b,m/n) or (m/n,c/d), making m/n the simplest fraction in the interval (a/b,c/d). Geoff goes on to prove that in fact the *mediant* of a/b and c/d, or (a+c)/(b+d), is adjacent to both a/b and c/d. (Note that 8/13 is the mediant of 3/5 and 5/8.) **Robert Orme '05** also found the same result, without proof.

Problem of the Week

Using the Law of Cosines, one can see that the 3-5-7 triangle has an angle of arccos(-1/2), or 120 degrees. What is curious is that there are integer-sided triangles which get closer and closer to right triangles: that is, with largest angle arccos(-1/3), arccos(-1/4), arccos(-1/5),

... (approaching arccos(0), a right angle). Find as many of these as you can and if possible, prove that there is an integer-sided triangle with angle arccos(-1/n) for any positive integer n.

** Please submit all solutions to David Molnar (molnar@stolaf.edu) by noon on Sunday.

If you would like to receive a copy of the Math Mess in your P.O. Box weekly, please e-mail Donna Brakke at brakke@stolaf.edu.

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