MSCS Mess

Department of Mathematics, Statistics and Computer Science
St. Olaf College
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This Week's Colloquium

Title:	Biostatistics and the Coin Toss: the Role of Randomization in Large Medical Studies
Speaker:	Roderick Little, Department of Biostatistics, University of Michigan
Time:	Friday, November 4, 3:40 p.m. (treats at 3:30)
Place:	SC 182

Biostatisticians love to toss coins, but now they use computers to do so! Tosses are used in randomization, a key statistical idea for the design of experiments and surveys. Randomization has two important roles in medical studies: it is used both for the allocation of treatments and for the selection of cases. Professor Little will discuss these roles in the context of two of the largest medical studies ever undertaken: the Salk vaccine field trial of 1954, and the National Children's Study (currently in its planning phase). In the former, the random allocation of treatments was the focus of controversy; in the latter, it was the random selection of participants that was hotly debated.

To Be or Not to Be

The MSCS Department is hosting its annual "To Be or Not to Be (A Math/Stats/CS Major or Concentrator)" gala this Tuesday, November 1. A faculty panel will discuss career paths in mathematics, show you which courses go well with other majors, and introduce you to different parts of the MSCS department (stats, CS, math education). There will be time for questions afterwards.

The gala begins at 6:30 p.m. in SC 282, but plan to arrive by 6:00 p.m. for pizza and root beer floats!

Attend a Pi Mu Epsilon Conference

We are looking for three students to attend the Pi Mu Epsilon Conference at St. Norbert College on November 4 and 5. Students will travel with Professor Humke and Taylor and three student speakers. We will depart around 11 a.m. on Friday, November 4, and return around dinnertime on Saturday, November 5. Housing, dinner on Friday and transportation are all provided.

If you are interested in attending the conference, please write a short paragraph on why you wish to attend and e-mail it to Amelia Taylor at ataylor@stolaf.edu by 5 p.m. on Wednesday, November 2. The first three students who send thoughtful responses will get to go, so send those responses early! (There may be some possible gender restrictions for housing reasons.)

Problem of the Week (POW)

Find all solutions of the equation x^y = y^x for real numbers x, y > 0.

*** Please submit all solutions by Wednesday at noon to Amelia Taylor by e-mail (ataylor@stolaf.edu) or by placing them in her box at OMH 201.

Announcing Spring 300-Level Courses

Student mathematicians have the opportunity to take two 300-level math courses this spring. The courses are offered at different times, so if you can't decided which sounds more appealing, you might as well take both…

Math 384: Applied Algebra

Now that you have studied the theory of groups and rings in Abstract Algebra, you can apply that knowledge to biology and cryptography in a project-based class. The goals of this course are twofold: first, to apply the theory learned in abstract algebra to problems from other fields of study, and second, to experience mathematical research. No prior experience with research, biology, or cryptography is necessary. Students will learn about doing research and what is needed from the other fields as part of the course. Prerequisite: Math 252, Abstract Algebra

If you have any questions about the course, please contact Amelia Taylor (ataylor@stolaf.edu)

Math 364: Combinatorics

Have you ever wanted to know exactly how many ways there are to run up the stairs if you can climb either one or two stairs at a time? Have you ever wanted to know 66 different ways to describe one sequence of integers? Or have ever just wanted to solve a problem in a 300-level class by drawing a picture? If so, you should register for Combinatorics.

Combinatorial ideas can be found in computer science, cryptography, physics and many other sciences. In this course we will discuss what it means to "count" things. Counting techniques will include inclusion-exclusion principle, generating functions and recurrence relations. We will also introduce some of the basics of graph theory and touch on codes and designs. Prerequisite: Math 252, Abstract Algebra

If you have any questions about the course, please contact Tina Garrett (garrettk@stolaf.edu).

Math Practicum Interviews

Interviews for this coming interim's Math Practicum (Math 390) will begin Monday, October 31, and continue through Thursday, November 10. A sign-up sheet can be found outside Professor Roback's office (OMH206), and interviews will be conducted in Professor Richey's office (OMH301). Please sign up if you are at all interested!!

The purpose of these short (15 minute) interviews is (a) to share with you expectations about the month of January, (b) to gauge your mathematical (and non-mathematical) strengths and interests, (c) to help us make decisions if we start creeping above the course enrollment limit of 15 students (5 per group), (d) to allow you to ask any questions you have about the Practicum, and (e) to get you fired up for what promises to be an intense but immensely interesting and satisfying month.

We are in the process of finalizing the three projects for this interim, but the projects are looking especially enticing! The tentative list of clients includes a medical device company, a high tech engineering manufacturer, and a non-profit environmental group. There is no specific mathematics background required for any project, just enthusiasm and an open, creative, analytical mind. Priority will be given to seniors, although there may be spaces available for interested juniors. If you have any questions, please contact Professors Richey or Roback.

Last Week's Problem of the Week

Select two integers (without replacement) from the set of integers between one and one million (inclusive). Is the sum of the two integers more likely to be even, be odd, or will the two outcomes be equally likely?

A challenge extension: Choose two rational numbers between zero and one (again without replacement, and inclusive of the endpoints). Is their sum, when written in reduced form, more likely to have an even numerator, odd numerator, or will the two outcomes be equally likely?

Congratulations to Thomas McConville '09, Jeremy Gustafson '08 and Paul Tveite '07 for solving this week's POW. The extension is still open!

The probability that the sum will be odd is slightly bigger. There are 500,000 even integers between 1 and one million (inclusive) and the same number of odd integers. The probability that the first number chosen is odd or even is $\frac{1}{2}$. But then the probability that a number of the same parity is chosen for the second number is $\frac{499{,}999}{999{,}999}$ (we have 1 less choice from a set from 1 less number of numbers) and the probability that a number of opposite parity is chosen is $\frac{500{,}000}{999{,}999}$ (all the opposite parity numbers are still in the pot). For the sum of two integers to be even the numbers must be of the same parity and thus this happens with slightly less probability.

Solution to the Airplane POW

As promised, this week we bring you the solution to the airplane problem from the issue before last.

Here is the question again: An airplane has exactly 80 passenger seats. Eighty passengers are in a single line to get on board, and each has a reserved assigned seat. Each passenger gets on one at a time to select his or her seat. However, the first passenger does not look at the assigned seat number and randomly selects a seat on the plane. Thereafter, each passenger takes his or her assigned seat - if it is already taken, the passenger selects another unoccupied seat at random. What is the probability that the last passenger on board will be able to sit in his/her originally assigned seat?

The following solution was submitted by Thomas McConville '09:

We find the probability that the last passenger does not get his/her originally assigned seat and it follows that the answer to the question above is 1 minus that probability. The last passenger does not get his/her seat if someone takes it before him/her. We can add the probabilities that person *i*, $1 \le i \le 80$, will take person 80's seat because there is no overlap; person *i* may only take person 80's seat as long as person *i*'s seat was already taken, except for $i = 1$. Let P_n be the probability that person n takes person 80's seat. Then

$P_1 = \frac{1}{80}$

$P_2 = \frac{1}{80} \times \frac{1}{79}$

$P_3 = \left(\frac{1}{80} \times \frac{1}{78}\right) + \left(\frac{1}{80} \times \frac{1}{79} \times \frac{1}{78}\right)$

$= (P_1 + P_2) \times \frac{1}{78}$

Note that this follows because the probability that person 1 or 2 takes person 80's seat is the same as the probability that person 1 or 2 takes person 3's seat, and the probability that person 3 takes person 80's seat is the product of the probability that person 2's seat is taken and the probability that person 2 takes person 80's seat. Similar logic argues the following:

$P_4 = (P_1 + P_2 + P_3) \times \frac{1}{77}$ and

$P_n = (P_1 + \ldots + P_{n-1}) \times \frac{1}{80-(n-1)}$

Factoring a bit, we find:

$$P_n = \frac{1}{80}\left(1+\frac{1}{79}\right)\left(1+\frac{1}{78}\right)\ldots\left(1+\frac{1}{80-(n-2)}\right)\left(\frac{1}{80-(n-1)}\right)$$

Then the full sum is:

$$\frac{1}{80} + \left(\frac{1}{80}\times\frac{1}{79}\right) + \frac{1}{80}\left(1+\frac{1}{79}\right)\frac{1}{78} + \frac{1}{80}\left(1+\frac{1}{79}\right)\left(1+\frac{1}{78}\right)\frac{1}{77}$$

$$\ldots + \frac{1}{80}\left(1+\frac{1}{79}\right)\left(1+\frac{1}{78}\right)\ldots\left(1+\frac{1}{3}\right)\left(\frac{1}{2}\right)$$

$$= \frac{1}{80}\left(1+\frac{1}{79}+\left(1+\frac{1}{79}\right)\frac{1}{78}+\ldots+\left(1+\frac{1}{79}\right)\left(1+\frac{1}{78}\right)\ldots\left(1+\frac{1}{3}\right)\frac{1}{2}\right)$$

$$= \frac{1}{80}\left(1+\frac{1}{79}\right)\left(1+\frac{1}{78}+\ldots+\left(1+\frac{1}{78}\right)\ldots\left(1+\frac{1}{3}\right)\frac{1}{2}\right)$$

$$= \ldots = \frac{1}{80}\left(1+\frac{1}{79}\right)\left(1+\frac{1}{78}\right)(\ldots)\left(1+\frac{1}{4}\right)\left(1+\frac{1}{3}+\left(1+\frac{1}{3}\right)\frac{1}{2}\right)$$

$$= \frac{1}{80}\left(1+\frac{1}{79}\right)\left(1+\frac{1}{78}\right)(\ldots)\left(1+\frac{1}{3}\right)\left(1+\frac{1}{2}\right)$$

$$= \frac{1}{80}\left(\frac{80}{79}\right)\left(\frac{79}{78}\right)(\ldots)\left(\frac{4}{3}\right)\left(\frac{3}{2}\right) = \frac{1}{2}$$

Hence, the probability that the passenger does get his or her seat is $\frac{1}{2}$.

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