MSCS Mess

Department of Mathematics, Statistics and Computer Science
St. Olaf College
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This Week's Colloquium

Title:	Counting, Recounting and q-Counting Fibonacci Identities
Speaker:	Tina Garrett, St. Olaf College
Time:	Tuesday, February 28, 1:30 p.m. (treats at 1:15)
Place:	SC 182

Fibonacci numbers and identities appear in many branches of mathematics. In this talk, Professor Garrett will explore a counting technique that relates domino tilings to Fibonacci numbers and will prove several interesting results. She will also consider weighted tilings and explain the concept of q-counting and q-Fibonacci numbers.

Tina Garrett did her undergraduate work at MIT in mathematics, graduating in 1994 with an SB. She earned her Ph. D. from the University of Minnesota in 2001. She taught at Carleton for four years before coming to St. Olaf in Fall 2005. Her research interests include partition theory, q-series, and special functions.

Problem of the Week (POW)

A little number theory this week:

Suppose p, q, and r are positive integers no two of which have a common factor larger than 1. Suppose P, Q, and R are positive integers such that $\frac{P}{p}+\frac{Q}{q}+\frac{R}{r}$ is an integer. Prove that each of $\frac{P}{p}, \frac{Q}{q}$ and $\frac{R}{r}$ is an integer.

*** Please submit all solutions by Wednesday at noon to Amelia Taylor by e-mail (ataylor@stolaf.edu) or by placing them in her box at OMH 201.

Last Week's Problem

In a particular game (not Yahtzee), you roll 5 dice twice. What is the probability of getting 5 of a kind? Assume that you are allowed in between the rolls to choose how many to re roll. Also, assume that if a pair is rolled the first time, that pair is kept and only 3 dice are re rolled. Finally, assume the dice are fair.

The solution given below is similar to the solution submitted by Kai Maeda, who was a visiting student in '92. Partition the sample space into the following 5 mutually exclusive events:
X_5 = The first roll all 5 are the same;

X_4 = 4 are the same on the first roll;
X_3 = 3 are the same on the first roll;
X_2 = there are either 1 or 2 pairs, but no 3 are the same;
X_1 = none are the same.

If E is the event that all 5 are the same after 2 rolls, then we can compute the probability as

$$\sum_{i=1}^{5} P(X_i)P(E \mid X_i),$$

where $P(E \mid X_i)$ is the conditional probability that E happens given that X_i already happened. For the first roll the sample space has 6^5 possible rolls.

Thus, $P(X_5) = \frac{6}{6^5}$, $P(X_4) = \frac{\binom{5}{4}\times 6\times 5}{6^5}$,

$$P(X_3) = \frac{\binom{5}{3}\times 6\times 5\times 5}{6^5},$$

$$P(X_2) = \frac{\binom{5}{2}\times\binom{3}{2}\times 6\times 5\times 4 + \binom{5}{2}\times 6\times 5\times 4\times 3}{6^5},$$

and $P(X_1) = \frac{6\times 5\times 4\times 3\times 2}{6^5}$.

For each $P(E|X_i)$ the number we are trying for has now been fixed, as we are given that one roll has happened. So the probabilities here are just 1 over 6^i where i is the number of dice we are rolling. This gives a total probability of

$$\left(\frac{6}{6^5}\right)(1) + \left(\frac{\binom{5}{4}\times 6\times 5}{6^5}\right)\left(\frac{1}{6}\right) + \left(\frac{\binom{5}{3}\times 6\times 5\times 5}{6^5}\right)\left(\frac{1}{6^2}\right)$$

$$+\left(\frac{\binom{5}{2}\times\binom{3}{2}\times 6\times 5\times 4 + \binom{5}{2}\times 6\times 5\times 4\times 3}{6^5}\right)\left(\frac{1}{6^3}\right)$$

$$+\left(\frac{6\times 5\times 4\times 3\times 2}{6^5}\right)\left(\frac{1}{6^4}\right) = .0137.$$

POW Archives

The following POW was printed in the November 14, 2005 issue of the MSCS. One solution was printed in the December 5, 2005 issue, but I promised to print a shorter solution if I received one. The following solution was submitted by both Kay Smith and Thomas McConville '09.

Let us assume that a given pair of people either know each other or are strangers. If six people enter a room, show that there must be either three people who know each other pairwise, or three people who are pairwise strangers.

We use an edge coloring argument to solve this POW. Make a complete (fill in all possible edges) graph on 6 vertices; color the edges between people who know each other red and the edges between people who do not know each other blue. Each vertex is part of 5 edges, as the graph has all possible edges. Then 3 of these edges must be all red or all blue. Let a_1, a_2, and a_3 denote the vertices that are the other endpoints for the 3 edges of the same color. If any of the edges between these vertices are the same color as the 3 edges, then we have a triangle of people who either pairwise know each other or pairwise do not. If none of the 3 edges are that color, then all three edges connecting a_1, a_2, and a_3 are the same color and we again get a triangle showing pairwise knowing or not knowing.

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