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Next Monday's Colloquium

Title:	The Circle Squaring Problem: A Tale of Undergraduate Research
Speaker:	Pam Pierce
Time:	Monday, October 10, 3:30 pm
Place:	RNS 310

About the talk: Tarski's famous Circle-Squaring problem asks whether a circle (a closed disk in the plane) can be decomposed into finitely many pieces which can then be rearranged to form a square. A definitive answer to this question eluded mathematicians for hundreds of years, until Miklos Laczkovich proved (in 1990) that it is indeed possible. While the answer was celebrated as a great victory, his proof gives no specific directions on how to accomplish the task. Furthermore, the upper bound given for the number of pieces required in the process is 10^{50}. In this talk we will learn some facts about dissections, decompositions, and what Laczkovich's "pieces" actually look like in an effort to fully understand the depth of this problem.

About the speaker: Pam Pierce received her B.A. degree in 1985 from Amherst College and her Ph.D. in Mathematics in 1994 from Syracuse University. Since that time she has been working at The College of Wooster, where she is currently a professor of mathematics. She has also served as an Associate Dean for Academic Advising and as Chair of the Department of Mathematics and Computer Science. Pam's primary research interests are in the field of real analysis, but she has advised student research projects in a variety of different fields. In this talk she will be sharing with us the details of one such student project, on Tarski's famous circle-squaring problem. An article on these results was published in *Math Horizons* in 2009 and was the recipient of the MAA's Trevor Evans award. In her spare time, Pam enjoys traveling, swimming, choral music, and taking walks with her dog Sean.

Next Thursday's Research Seminar

Title:	Northfield Undergraduate Research Symposium
Speakers:	Various Oles and Carls!
Time:	Thursday, October 13, 3:40-6:50 pm
Place:	Carleton CMC, 3$^{\text{rd}}$ Floor

About the event: Each year, students from St. Olaf and Carleton have the opportunity to present their own research (conducted in Northfield or at campuses nationwide) to peers and faculty members at the Northfield Undergraduate Mathematics Symposium. This year, five students from St. Olaf and three from Carleton will give a total of six twenty-minute talks, with time for food and socialization in-between. Abstracts for the presentations are provided below.

Daniel Lewitz (Carleton), "m-gapped Progressions and van der Waerden Numbers."

An m-gapped progression is a generalization of an arithmetic progression in which the gaps in the progression only need to belong to a set of m elements, rather than all be the same. In the same way that arithmetic progressions pertain to the van der Waerden numbers, $W(k;r)$, m-gapped progressions pertain to a function we call $B_m(k;r)$. We will examine some results about the nature of the function $B_m(k;r)$, both in general and for special case when $r = 2$. In particular, we will show how there are exact results for $B_m(k;r)$ when k is relatively small. This work was done with Catherine Cooper, Trinity College; Alex Stoll, Clemson University; and Bruce Landman, University of West Georgia.

Conrad Parker and Melanie Stevenson (St. Olaf), "An Involution Proof of a Borwein Theorem for Overpartitions"

In 1990, P. Borwein conjectured a $+--$ sign pattern for polynomials counting certain signed integer partitions. We conjecture a $+-$ 0 pattern for the generating function for overpartitions into parts not divisible by 3 and give an involution-based proof of the 0 case of this conjecture using pentagonal numbers and the Jacobi triple product. We also share a proof of a generalization of this case involving quadratic nonresidues modulo a prime.

Eleanor Campbell (Carleton), "A Combinatorial Model of Quantum Skew Symmetric Matrices"

The quantized coordinate ring of $m \times n$ Quantum matrices, or simply quantum matrices, holds deep connections to the theory of totally nonnegative matrices, wave interactions and knot theory. We examine the less understood theory of quantum skew-symmetric matrices $O_q(Sk_n)$ over a field k. This algebra is known to be generated by a set of generators $y_{i,j}, 1 \leq i < j \leq n$, which satisfy certain commutativity relations dependent on some element $q \in k$. We view $O_q(Sk_n)$ from a combinatorial perspective. We prove $O_q(Sk_n)$ is isomorphic to an algebra called A_n over k, defined graphically. A_n is generated by elements x_{ij}, where each x_{ij} is the sum of the weights of paths from i to j in a particular directed graph. The weights are obtained from elements of a space with simpler commutativity relations dependent on q. Using inductive methods on the graph, we prove that the generators of A_n satisfy the same commutativity relations of $O_q(Sk_n)$, allowing for a new combinatorial perspective that may be used to study this algebra.

Quinton Neville (St. Olaf), "Partial Differential Modeline in the Kidney"

The kidney, a diverse and complicated biological system, is most simply charged with the production of urine. At a deeper level, the functional unit that facilitates this process is the nephron. The nephron utilizes the Tubuloglomerular Feedback (TGF) System to monitor chloride levels during the production of urine, making sure the body is not retaining or losing too much. In mammals, there are two types of nephrons: short-looped and long-looped. The key difference between the short-loop and the long-loop is the existence of the Thin Ascending Limb (THAL) in the long-loop, which has differing properties for spatially varying permeability and maximum transport rate of chloride. Additionally, there is biological evidence of an association between desert mammals' ability to produce more highly concentrated urine and a higher percentage of long-looped nephrons in their kidneys. The long-looped nephron, however, is not well characterized biologically, which provides motivation to attempt to explain this association mathematically. Thus, we developed a partial differential model of a long-looped nephron, derived a characteristic ordinary differential equation, varied the length of the model THAL, and performed a bifurcation analysis of the long-looped TGF system. Our analysis indicates that there is a higher tendency towards oscillatory solutions in the long-looped TGF system than in the short-looped, and increasing the length of the THAL may create a more stable TGF system.

Abdel-Rahman Madkour and Philip Nadolny (St. Olaf) *"Finding Minimal Spanning Forests in a Graph"*

In the computation of multidimensional persistent homology, a popular tool in topological data analysis, a family of planar graphs arises. We have studied the problem of partitioning these graphs in a way that will be useful for parallelizing the persistent homology calculation. Specifically, we desire to partition an edge-weighted, undirected graph G into k connected components, $G_1, \ldots, G_k$. Let w_i be the weight of a minimum spanning tree in component G_i. For our purposes, an ideal partition is one that minimizes $\max w_1, ..., w_k$. This problem is known to be NP-hard in the case of general graphs and we are unable to find this specific problem in the graph partitioning literature. We propose two approximation algorithms, one that uses a dynamic programming strategy and one that uses a spectral clustering approach, that produce near-optimal partitions in practice on a family of test graphs. We present detailed descriptions of these algorithms and the analysis of empirical performance data.

Shatian Wang (Carleton), ''The Metric s-t path Travelling Salesperson Problem and the Randomized Christofides Algorithm"

In the well-known metric Traveling Salesman Problem (metric TSP), a complete graph G= (V, E) is given with nonnegative metric edge costs. The goal is to find a Hamiltonian circuit in G with minimum cost. The Christofides heuristic (1976) gives a nice purely combinatorial 1.5-approximation algorithm for the metric TSP. An important variant of the metric TSP is the metric s-t path TSP, in which two fixed vertices, s and t are given, and the goal is to find a min-cost Hamiltonian path from s to t. The Christofides heuristic can be easily extended to this s-t path variant, but only with an approximation ratio of 5/3. An, Kleinberg and Shmoys (2012) made the first improvement to 5/3 with an LP based algorithm, the randomized Christofides algorithm, and achieved an approximation ratio of 1.618. This LP based analysis has inspired a sequence of later breakthroughs, including an 1.6 bound by Sebo (2013) and an 1.566 bound by Gottschalk and Vygen (2016). If time permits, a more general problem, the connected T-join problem, will be discussed at the end of the talk.

Math in the news

In what must be an astonishing week for our MSCS/physics double-majors and pure mathematicians alike, physicists David Thouless, F. Duncan Haldane, and J. Michael Kosterlitz have won the Nobel Prize in Physics for work that related topological concepts to previously misunderstood phases of matter. Inspiring!

A solution to last week's problem

Last week we posed the theorem that among any five points in the plane (with no three collinear), there re four points that re the vertices of a convex quadrilateral. We hope you enjoyed thinking about this problem, and we now present the Klein's original explanation for why the theorem holds.

Take five points in the plane with no three collinear. Now, consider the convex hull of those points, which is the smallest convex shape in the plane that encloses all five points. If four of your points are vertices of the convex hull, those four points form a convex quadrilateral, which is lucky. Now, we consider the only other possible case, that three of your points are vertices of the convex hull and the other two lie inside the hull. But then the two inner points form a convex quadrilateral with one of the sides of the triangular convex hull (can you see why?), and we are done.

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